

# **Geostatistics and petroleum geology**

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Second edition

**Michael Edward Hohn**

*West Virginia Geological Survey*



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Typeset by Photoprint, Torquay, Devon.

To my wife, Kay,  
and my children, Geoffrey and Abigail

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# Preface

This is an extensive revision of a book that I wrote over ten years ago. My purpose then has remained unchanged: to introduce the concepts and methods of spatial statistics to geologists and engineers working with oil and gas data. I believe I have accomplished more than that; just as I learned the basics of variography and kriging from books for mining engineers, this book could be used by scientists from many fields to learn the basics of the subject.

I have tried to adopt an introductory and practical approach to the subject, knowing that books that detail the theory are available. What I say and write comes from my own experience. As a geologist working in the public sector, I have had the privilege of using geostatistics in funded research, in answering service requests from industry, and in short courses. I have taught geostatistics in the university classroom, and advised graduate students in theses and dissertations. I have attempted to anticipate the needs and questions of the enquiring scientist because I was there myself, and know the kind of questions and concerns I had at the time I was trying to learn the subject.

Geostatistics has become a toolbox of methods useful for attacking a range of problems. The number of methods is now broad enough – even for such an established method as kriging – that determining the best one for all situations has been largely abandoned. One still finds comparative studies, but most leave the impression that the efficacy of a particular method varies with the data. Hence, I have avoided direct comparison of techniques, preferring instead to illustrate features of each. In general, the geostatistician should learn a range of methods, and pick the one that works best for a given situation, given available software or a willingness to write computer programs.

Necessary items in the geostatistical toolbox of today are the ability to calculate and model semivariograms, linear methods of kriging, cokriging and variants of cokriging, nonlinear methods such as indicator kriging or disjunctive kriging, and conditional simulation. All geostatistics depends on a model of spatial dependence, hence variography. Even nonlinear geostatistical estimation methods, and most methods of simulation require setting up and solving the systems of equations used

in linear kriging. Indicator approaches to estimation and simulation are used widely. Although disjunctive kriging is encountered less often, it uses that important technique, the computation of normal scores. There has been a virtual explosion in methods and application papers on conditional simulation (or stochastic simulation) in the ten years since my original book came out. Conditional simulation has become important in the area of reservoir characterization, an area where the geologist and engineer can cooperate effectively.

To learn any method, one needs to know the motivation for the method, how it works, the steps the user must go through, and problems the user might encounter. The best way to do all this is through real examples. I have placed a strong emphasis on the examples; most of the time spent on this revision was on developing new examples to reflect where geostatistics has gone in the past ten years. I have elected to use real data sets, with all the ensuing problems, and I have 'let the warts show'. Some of the examples did not turn out as well as I expected, but they had the advantage of being data that I was familiar with because in most cases they came out of projects that I worked on.

Geostatistics is by nature mathematical and statistical. I have not tried to avoid mathematics when it seemed necessary; the educated user should be exposed to the mathematical underpinnings of any method. No advanced mathematical knowledge is required, and in places I have tended to simplify where possible. The emphasis throughout is on what the practitioner needs to know, and the results one can expect to get.

I wish to thank Ronald R. McDowell, Senior Research Geologist at the West Virginia Geological Survey, for help in programming and for reading the manuscript. Raymond Strawser drafted some of the figures. Richard Chambers provided data used in Chapter 4. Frank Curriero fielded some of my statistical questions. This work would not have been possible without the support and encouragement of Larry D. Woodfork, Director and State Geologist, and Carl J. Smith, Deputy Director, of the West Virginia Geological and Economic Survey.

Michael Edward Hohn  
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